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On realizing the bosonic string as a noncritical W_3-string

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Abstract

We discuss a realization of the bosonic string as a noncritical W_3-string. The relevant noncritical W_3-string is characterized by a Liouville sector which is restricted to a (nonunitary) (3,2) W_3 minimal model with central charge contribution $c_l = -2$. Furthermore, the matter sector of this W_3-string contains 26 free scalars which realize a critical bosonic string. The BRST operator for this W_3-string can be written as the sum of two, mutually anticommuting, nilpotent BRST operators: $Q = Q_0 + Q_1$ in such a way that the scalars which realize the bosonic string appear only in Q_0 while the central charge contribution of the fields present in Q_1 equals zero. We argue that, in the simplest case that the Liouville sector is given by the identity operator only, the Q_1-cohomology is given by a particular (nonunitary) (3,2) Virasoro minimal model at $c = 0$.

1. Introduction

It is well-known that the Virasoro algebra is the underlying symmetry algebra of string theory. Extensions of the Virasoro algebra, including generators of spin higher than two, are generically called W-algebras. The simplest example is the so-called W_3-algebra which contains an additional generator of spin three [1]. They can be used to construct new string theories, the so-called W-strings [2].

A systematic investigation of multi-scalar matter free-field realizations of the W_3-algebra was undertaken in [3]. In particular, in [3] a general mechanism was presented for generating realisations of the W_3-algebra, given any realization of the Virasoro algebra, by adjoining an extra matter scalar field, say ϕ_2. All other scalars occur via their energy-momentum tensor, say T_μ. The different scalars contribute to the matter central charge $c_m = c_\mu + c_{\phi_2}$ as follows:

$$c_\mu = \tfrac{1}{4}c_m + \tfrac{1}{2}\,, \quad c_{\phi_2} = \tfrac{3}{4}c_m - \tfrac{1}{2}\,. \tag{1}$$

Note that the critical value $c_\mu = 26$ of the Virasoro algebra does not lead to the critical value

$$c_m = 100 \tag{2}$$

of the W_3-algebra. Instead it leads to the anomalous value $c_m = 102$. In [4] it was pointed out that this excess of two units of central charge could be compensated by passing from a "critical" to a "noncritical" W_3-string[1] with a Liouville sector which is restricted to the (nonunitary) (3,2) model with central charge contribution $c_l = -2$. The critical value of the central charge is now given by

$$c_m + c_l = 100 \tag{3}$$

which is indeed satisfied by the values $c_m = 102$ and $c_l = -2$. Based on the above numerology, it was

[1] Using the terminology of [5], a "noncritical" W_3-string is characterized by the fact that there is a matter and Liouville sector which *separately* satisfy a W_3-algebra. If the Liouville sector is absent we call the corresponding model a "critical" string theory.

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suggested in [4] that the above noncritical W_3-string should in some sense contain a critical Virasoro string with 26 free scalars.

After the work of [4] it was shown [6] that, by going to a new basis in the Hilbert space, the BRST operator of the W_3-algebra can be written as the sum of two, mutually anticommuting, nilpotent BRST operators

$$Q = Q_0 + Q_1 . \tag{4}$$

It was subsequently shown in [7] that if one chooses for the Liouville sector a (p,q) W_3 minimal model then the cohomology of the Q_1-operator is given by a (p,q) Virasoro minimal model. For instance, the special case of a $(4,3)$ W_3 minimal model with central charge $c_l = 0$ leads to a $c_1 = 1/2$ Ising model in the Q_1-cohomology [2]. In view of possible connections with the bosonic string we are particularly interested in the case that the Q_1-cohomology is given by a (unitary) $(3,2)$ Virasoro minimal model at $c_1 = 0$ which corresponds to just one state with conformal weight $h_1 = 0$. The complete Q-cohomology is then identical to that of a critical Virasoro string. Unfortunately, the arguments of [7] are only valid for values of (p,q) with $(p,q) \geqslant (4,3)$. For these values there is a Kac-table characterizing the finite set of primary fields of the model.

It is the purpose of this letter to investigate the extrapolation of the results of [7] to values of (p,q) with $(p,q) < (4,3)$. Having the connection with the bosonic string in mind we will consider the particular case $(p,q) = (3,2)$. We will argue that if one chooses for the Liouville sector a (necessarily nonunitary) W_3 minimal model [3] at $c_l = -2$ then the Q_1-cohomology is given by a corresponding nonunitary $(3,2)$ Virasoro minimal model at $c_1 = 0$. We will only give explicit results for the simplest case that the W_3 minimal model is given by the identity operator. The complete Q-cohomology contains, among other states, all the states of a critical bosonic string.

[2] A similar result for the critical string was established earlier in, e.g., [8–10].

[3] We will call a (p,q) (W_3-)model *minimal*, if it is realized by a finite number of (W_3-)primaries which form a closed operator product algebra. We infer the closure of the operator algebra from the fusion rules à la BPZ [11]. This does not imply that the nonunitary minimal models considered here necessarily respect modular invariance. We thank Jan de Boer for a discussion on this point.

Table 1
The fields of the noncritical W_3-string. The scalar fields of the matter sector (X^μ, ϕ_2) and of the Liouville sector (σ_1, σ_2) are given with their background charge, and their contribution to the central charge. The fields (c, b) are the spin-2, (γ, β) the spin-3 ghosts.

Field	Background charge	Central charge
X^μ	0	26
ϕ_2	$\frac{5}{2}$	76
σ_1	$-\frac{1}{6}i\sqrt{3}$	0
σ_2	$-\frac{1}{2}i$	−2
c, b		−26
γ, β		−74

2. The cohomology

In this section we will calculate the Q_1-cohomology at level zero and level one. In Table 1 we present the fields of the model with their background and central charges. Note that the matter sector contains 26 free scalars X^μ without a background charge. To facilitate the comparison with [7] we have used a two-scalar realisation of the Liouville sector.

In [5,12] the BRST-operator for this system was calculated. It is given by $Q = \oint j$, with $j = j_0 + j_1$ given by

$$j_0 = c\,\{T_M + T_L + T_{(\gamma,\beta)} + \tfrac{1}{2}T_{(c,b)}\}\,, \tag{5}$$

$$\begin{aligned} j_1 = \gamma\,[\, & \frac{i}{3\sqrt{6}}\{4(\partial\phi_2)^3 - 30\partial\phi_2\partial^2\phi_2 + 10\partial^3\phi_2\} \\ & + i\{W_L - \frac{2}{\sqrt{6}}\partial\phi_2 T_L + \frac{5}{2\sqrt{6}}\partial T_L\} \\ & - i\sqrt{6}\{\partial\phi_2\partial\gamma\beta + \tfrac{5}{6}\partial\beta\partial\gamma\}\,]\,. \end{aligned} \tag{6}$$

$Q_0 = \oint j_0$ and $Q_1 = \oint j_1$ are seperately nilpotent, and therefore anticommute. Note that the scalars X^μ, and the spin-2 ghosts are absent from j_1, and that the Liouville fields do not occur explicitly in (6), but only via the generators T_L and W_L which represent a standard two-scalar realization of the W_3-algebra. In j_0 we find, besides T_L, the energy-momentum tensors of the matter and ghost sectors:

$$T_M = -\tfrac{1}{2}(\partial X^\mu)^2 - \tfrac{1}{2}(\partial\phi_2)^2 + \tfrac{5}{2}\partial^2\phi_2\,, \tag{7}$$

$$T_{(c,b)} = -2b\partial c - (\partial b)c\,, \tag{8}$$

$$T_{(\gamma,\beta)} = -3\beta\partial\gamma - 2(\partial\beta)\gamma\,. \tag{9}$$

We first calculate all the states in the Q_1-cohomology at level zero. At level 0, the states of lowest ghost number $G = 2$ are of the form[4,5,6]

$$V_{0,2}(p_2, s_1, s_2) = (\partial\gamma)\gamma\, e^{ip_2\phi_2 + is_1\sigma_1 + is_2\sigma_2}\,. \tag{10}$$

The condition $Q_1 V_{0,2}(p_2, s_1, s_2) = 0$ determines the momenta of the three fields. The resulting cubic equation factorizes, and we obtain the following three solutions for p_2 and the corresponding conformal weight h_1 of V:

$$(A_0)\ p_2 = i(s_2 - 2)\,, \qquad h_1 = \tfrac{1}{2}s_1(s_1 + 1/\sqrt{3})\,, \tag{11}$$

$$(B_0)\ p_2 = -\tfrac{1}{2}i(s_2 - \sqrt{3}\,s_1 + 5)\,, \qquad h_1 = \tfrac{1}{8}(s_1 + \sqrt{3}\,(1+s_2))(s_1 + (1+3s_2)/\sqrt{3})\,, \tag{12}$$

$$(C_0)\ p_2 = -\tfrac{1}{2}i(s_2 + \sqrt{3}\,s_1 + 6)\,, \qquad h_1 = \tfrac{1}{8}(s_1 - \sqrt{3}s_2)(s_1 - (2+3s_2)/\sqrt{3})\,, \tag{13}$$

where the Liouville momenta s_1 and s_2 are arbitrary. We thus find a two-parameter continuous spectrum of physical states at level zero.

It is not surprising that without any restriction on the Liouville sector we find a continous spectrum. The same happens for the noncritical W_3-strings studied in [7]. In [7] a discrete spectrum could be obtained by restricting the Liouville sector, by hand, to a W_3 minimal model. In the present case we cannot impose a similar restriction since the Kac table corresponding to the $(p,q) = (3,2)$ Liouville sector is empty. However, in a first stage, it is consistent to constrain the Liouville sector to the completely degenerate representations, since these representations form a closed operator algebra [11,13]. Explicitly, the Liouville momenta can be put on the lattice

$$s_1 = -\tfrac{1}{6}\sqrt{3}(2r_2 - 3t_2)\,, \qquad s_2 = -\tfrac{1}{6}(2(2r_1 - 3t_1) + (2r_2 - 3t_2)) \tag{14}$$

for arbitrary non-negative integers (r_1, r_2, t_1, t_2). Substituting these Liouville momenta into the solutions (11)-(13), we find that the conformal weights h_1 occurring in the Q_1-cohomology, correspond to the completely degenerate representations of the $c = 0$ Virasoro algebra with conformal weights

$$h_{\rm Vir} = \tfrac{1}{24}((2r - 3t - 1)^2 - 1) \tag{15}$$

for arbitrary non-negative integers (r,t).

The Q_1-cohomology is now reduced from a continuous to a discrete, albeit infinite, spectrum. In [7] the infinite spectrum could be reduced to a finite spectrum by imposing further conditions involving the Kac table. In the present case no such Kac table exists. However, the Liouville sector can be restricted further if there exist subsets of the completely degenerate representations that form closed operator algebras.

At this point we may use the results of [14] where fusion rules for W_N algebra representations are given. Using these fusion rules, it turns out that also in the $c_l = -2$ case we can have closed fusion rules with a finite number of primaries. We will continue our analysis for the simplest case where we restrict the Liouville sector to the unit operator with zero conformal and spin-3 weights. Modulo translations, it has 6 representatives on the lattice which are connected to each other by a Weyl transformation. It turns out that four of these representatives disappear in the cohomology with respect to the screening operators of the $c_l = -2$ W_3-algebra. The remaining two representatives of the identity operator are given by $(0,0;0,0)$ and $(1,1;0,0)$. They are related to each other by conjugation in the Liouville momenta. Ignoring the cohomology with conjugate momenta we can restrict ourselves to the single point $(r_1, r_2; t_1, t_2) = (0,0;0,0)$ or $(s_1, s_2) = (0,0)$ on the Liouville lattice (14). This leads in (11)-(13) to the weights $h_1 = (0, \tfrac{1}{8}, 0)$.

One can repeat the same analysis at level one. Our results are summarized in Table 2. We obtain for ghost

[4] At level 0 it is enough to consider the states at lowest ghost number $G = 2$. The states of higher ghost number $G = 3$ can be obtained by acting with picture-changing operators.

[5] The level of a state in the Q_1-cohomology is defined by level $\equiv h+3$, where h is the weight of the fields in front of the exponential. In the case of the Q-cohomology (see below) we have level $\equiv h+4$.

[6] Our notation for the states is $V_{l,G}$ for the Q_1-cohomology and $U_{l,G}$ for the total cohomology. Here l is the level and G the ghost number.

Table 2
The conformal weight h_1 of the states in the Q_1-cohomology at level 0 and level 1. States related to those shown by picture-changing or conjugation of the momenta are not presented.

G	Level 0	Level 1
1	—	$(0, \frac{5}{8})$
2	$(0, \frac{1}{8}, 0)$	—

number $G = 1$ a new state with weight $\frac{5}{8}$. Other states in the Q_1 cohomology at this level are related to the $G = 1$ states by picture changing or conjugation [7].

This concludes our analysis of the Q_1-cohomology at level zero and level one. To obtain the total Q-cohomology we must dress the states found in the Q_1-cohomology with spin-2 ghosts and X^μ. At level zero and lowest ghost number $G = 3$ this gives the state

$$U_{0,3}(p_X, p_2, s_1, s_2) = c\,(\partial\gamma)\gamma\, e^{ip_X\cdot X + ip_2\phi_2 + is_1\sigma_1 + is_2\sigma_2}\,. \tag{16}$$

The condition that Q annihilates this state implies that Q_0 and Q_1 each annihilate this state [8]. Thus in the Q-cohomology we find again the momenta (11-13), as well as the condition that $Q_0\, U_{0,3}(p_X, p_2, s_1, s_2) = 0$. This condition tells us that the total conformal weight of the physical state should vanish, and determines the value of $(p_X)^2$. We find:

$$(p_X)^2 = 2(1 - h_1)\,, \tag{17}$$

where h_1 is given in (11-13).

From (17) we deduce that for $h_1 = 0$ we have the (dressed) tachyon of the critical Virasoro string with $(p_X)^2 = 2$. It is not too difficult to see that for $h_1 = 0$ all the states of the bosonic string occur. One just replaces the $e^{ip_X\cdot X}$ part of (17) by any bosonic string vertex operator of the form $P(\partial X^\mu, \partial^2 X^\mu, \cdots)e^{ip_X\cdot X}$. We thus conclude that any $h_1 = 0$ solution in the Q_1-cohomology leads to a bosonic string spectrum in the total Q-cohomology. Alternatively, one can argue on more general grounds that the bosonic string spectrum is contained in the total Q-cohomology. Note that the total BRST operator Q can be written as $Q = Q_{\rm Vir} + Q_R$, where $Q_{\rm Vir}$ is the standard BRST operator of the Virasoro string and Q_R represents all other terms in Q. Since Q_R does not depend on X^μ and b any bosonic string state of the form $cV(X^\mu)$ is automatically Q-invariant. Sofar, our explicit calculations indicate that such a state is never Q-trivial.

From the results given in Table 2 we conclude that there are more states in the total Q-cohomology then the bosonic string states. They correspond to the $h_1 \neq 0$ solutions. Since we have chosen in the Liouville sector a (nonunitary) W_3 minimal model at $c_l = -2$ consisting of the identity operator only one would expect that the operators in the Q_1-cohomology form a corresponding (nonunitary) Virasoro minimal model at $c_1 = 0$. In the next section we will present additional arguments in support of this.

3. Other methods

The explicit calculation of the Q_1-cohomology becomes increasingly complicated at higher levels. It is therefore instructive to apply other methods as well. In this section we will first apply a conjecture from [7] which is a formula summarizing the Q_1-cohomology in the unitary case [9]. Secondly, we will also apply the fusion rules for Virasoro representations at $c_1 = 0$.

(1) The formula of [7] gives a criterion for which values of the momenta (p_2, s_1, s_2) a solution to the Q_1-cohomology can be expected. This formula does not depend on the existence of a Liouville lattice and/or a Kac table and can be applied in our case as well. First, we define the polynomials P_1, $P_2^\pm$ and P_3 by [10]

$$P_1(n) = p_2 - is_2 + i(n+2)\,, \tag{18}$$

$$P_2^+(n) = p_2 + \tfrac{1}{2}is_2 - \tfrac{1}{2}is_1\sqrt{3} + \tfrac{1}{2}i(3n+5)\,, \tag{19}$$

$$P_2^-(n) = p_2 + \tfrac{1}{2}is_2 - \tfrac{1}{2}is_1\sqrt{3} + \tfrac{1}{2}i(2n+5)\,, \tag{20}$$

$$P_3(n) = p_2 + \tfrac{1}{2}is_2 + \tfrac{1}{2}is_1\sqrt{3} + \tfrac{3}{2}i(n+2)\,. \tag{21}$$

Restricting attention to Virasoro primaries the formula of [7] conjectures that the Q_1-cohomology can be

[7] If excitations of the Liouville fields $\sigma_{1,2}$ are included, we obtain at $G = 2$ two states with $h_1 = 1$, which are not descendants of the level 0, $h_1 = 0$ states. However, these $h_1 = 1$ states disappear if a Felder reduction of the free field realisation of the Liouville minimal model is performed. These states occur in the noncritical W_3-string for a (p, q) Liouville sector with arbitrary p and q.

[8] At higher levels this becomes more complicated.

[9] This conjecture is suggested by the analysis of [15]. A rigorous proof is still lacking.

[10] These polynomials are specific to the case $(p, q) = (3, 2)$.

organized as follows: [11] If integers $u_1, u_2 < 0$ exist such that (i) $P_1(u_1) = P_2^+(u_2) = 0$, (ii) $P_2^-(u_1) = P_3(u_2) = 0$, or (iii) $P_1(u_1) = P_3(u_2) = 0$ there are two states at level $u_1 u_2$ and ghost numbers 1, 2.

Applying this and restricting ourselves to the identity operator on the Liouville lattice, we reproduce all the states of Table 2.

As an example, consider the case that $P_1(u_1) = P_2^+(u_2) = 0$. This implies

$$p_2 = -i(2 + u_1 - s_2)\,,$$
$$s_1 = \frac{1}{\sqrt{3}}(1 - 2u_1 + 3u_2 + 3s_2)\,. \qquad (22)$$

Clearly, a solution for $(s_1, s_2) = (0,0)$ can only exist if $2u_1 - 3u_2 = 1$. Substituting this in the formula for the conformal weight, we find

$$h_1 = u_1 u_2 + \tfrac{1}{2}u_1(1 - u_1) \qquad (23)$$

for level $u_1 u_2$. This shows that in this particular case all states have integer conformal weight. Taking the other cases into account as well, we find that there are only states with weights $h_1 = n, \frac{1}{8} + n$ or $\frac{5}{8} + n$ for non-negative integers n in the Q_1-cohomology.

(2) For $(p,q) \geqslant (3,2)$ the fusion rules of the Virasoro algebra give rise to the usual Kac tables. For $(p,q) = (3,2)$ the Kac table only consists of the identity, but it is not difficult to see that there exist other choices of finite sets of primaries that are closed under the BPZ fusion rules. For instance, the fusion rules show that $\{[0], [\frac{1}{8}], [2], [\frac{1}{3}]\}$ form a closed set. At level 1 we found in the Q_1-cohomology a state with weight $\frac{5}{8}$. If we include this primary in the previous minimal set we obtain the following set of primaries that are closed under the fusion rules: $\{[0], [\frac{1}{8}], [2], [\frac{1}{3}], [\frac{5}{8}], [1], [-\frac{1}{24}]\}$. It seems that of these, the dimension $\frac{1}{3}$ and $-\frac{1}{24}$ operators do not occur in the Q_1-cohomology. The others do occur in the Q_1-cohomology together with their descendants [12]. It is however not inconsistent that we don't have dimension $\frac{1}{3}$ and $-\frac{1}{24}$ operators in the Q_1-cohomology. They may decouple from the fusion rules. In fact, using the relation between the conformal weight h_1, the level, and the momentum p_2 (for vanishing s_1 and s_2) one can see that the weight $\frac{1}{3}$ cannot occur in the OPE of two dimension $\frac{1}{8}$ operators. According to the fusion rules this would have been the place where dimension $\frac{1}{3}$ enters the algebra. With dimension $\frac{1}{3}$ absent, there is no longer any need for a dimension $-\frac{1}{24}$ operator.

The two points above, together with our explicit calculations, indicate that with the Liouville sector restricted to the identity operator, the Q_1-cohomology is given by a (nonunitary) Virasoro minimal model at $c_1 = 0$ characterized by the finite set of primaries $\{[0], [\frac{1}{8}], [\frac{5}{8}]\}$.

4. Conclusions

In this letter we have shown in which sense the bosonic string spectrum occurs in the spectrum of a noncritical W_3-string with $c_l = -2$. We have restricted our analysis to the simplest case that the minimal model is given by the identity operator in the Liouville sector. One could consider other minimal models as well. The next to simplest model consists of three primaries $\{1, \phi_k^+, \phi_k^-\}$, $k = 1, 2, 3, \ldots$, with spin-two weights $\{0, k(2k-1), k(2k-1)\}$, respectively [14]. We expect that this family of W_3 minimal models will lead to a corresponding family of (nonunitary) $c = 0$ Virasoro models in the Q_1-cohomology. It would be interesting to see if choices are possible which respect modular invariance.

We expect that our results can be extended to the general case of a critical W_{n-1} string realized as a noncritical W_N-string. The generalization of the Q_1-operator is a BRST operator Q_N^n corresponding to the subalgebra $v_N^n \subset W_N$ consisting of generators of spin $s = \{n, n+1, \cdots, N\}$ [13]. We now restrict the Liouville sector to a $(n, n-1)$ model of the W_N algebra with $(3 \le n \le N)$ and central charge

$$c_l = (N-1)\{1 - \frac{N(N+1)}{n(n-1)}\}\,. \qquad (24)$$

[11] We give here a simplified version of the formula in the sense that we do not give (i) the level zero result which we have already given in the previous section (it agrees with [7]), (ii) solutions which can be obtained by conjugation of the momenta and (iii) a class of possible extra states which occur in the formula of [7] but which do not play a role in the present discussion.

[12] We assume that [1] and [2] are (null-)descendants of the identity.

[13] We use the notation of [7].

In that case we find that the central charge contributions of all fields present in the Q_N^n BRST operator add up to zero. This means that the states in the Q_N^n-cohomology with zero weights lead to a critical W_{n-1}-string in the total cohomology. The special case of $n = N = 3$ corresponds to the situation described in this letter.

Finally, we would like to mention recent work on the embedding of a bosonic string into a "critical" W_3-string [16]. In this letter we have described the embedding of a bosonic string into a particular "noncritical" W_3-string. It would be interesting to see whether there is any relation with the "critical" case. We expect that such a relation, if it exists, will be nontrivial, since the critical and noncritical W_3-strings are based on different algebras.

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